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MODELING OF FLUID FLOW IN PIPELINE AND THE DIFFERENCE SCHEME STABILITY INVESTIGATIONS

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A mathematical model of viscous fluid flow in the pipeline with the presence of flow across the surface and the narrowing of pipeline section, which is based on a system of Navier-Stokes equations in two-dimensional rectangular region with a special type of boundary conditions has been designed. The geometric configuration of the leakage zone is taken into the account. It is believed that the fluid motion is under the influence of constant length of pressure difference. For the solving of this system, the numerical method of finite differences was developed by which the finite differences scheme is realized – the first step is implicit in longitudinal coordinate, and the second – on the transversal. The study on the stability of the spectral features method, stability conditions are established for the case of flow calculation with specified parameters and for the given type of the pipeline geometry. The criterion of numerical stability is presented taking to account the model's parameters.

Keywords: Navier-Stokes system, boundary conditions, pipeline's section narrowing, low leakage, numerical method, stability, spectrum characteristics of stability.

Побудовано математичну модель течії в'язкої рідини в трубопроводі за наявності перетікання рідини через поверхню та звуження поперечного перерізу, яка базується на системі рівнянь Нав'є – Стокса в двовимірній прямокутній системі координат зі спеціальним типом граничних умов. Враховано просторову конфігурацію зон перетікання. Рух рідини здійснюється під дією постійного перепаду тиску по довжині труби. Для розв'язання задачі використано метод скінчених різниць, розроблено чисельний метод його реалізації – перший крок ітераційного процесу здійснюється по повздовжній, другий – по поперечній координатах. Вивчення стійкості проводиться за спектральною ознакою, встановлено умови стійкості для розрахунку течії зі спеціальними параметрами і для заданого типу геометрії труби. Критерії стійкості розрахунків представлено з урахуванням параметрів моделі.

Ключові слова: система Нав'є – Стокса, граничні умови, звуження перерізу труби, перетікання рідини, чисельний метод, спектральна ознака стійкості.

Построено математическую модель течения вязкой жидкости в трубопроводе при наличии перетекания жидкости через поверхность и сужения поперечного сечения, основанная на использовании двухмерной системе уравнений Навье – Стокса в прямоугольной системе координат со специальным типом граничних условий. Учтена пространственная конфигурация зон перетекания. Движение жидкости обусловлено постоянным перепадом давления по длине трубы. Для решения задачи используется метод конечних різностей, создан численный метод его реализации – первуй шаг итерационного процесса осуществляется по продольной, второй – по поперечной координатах. Изучение устойчивости проводится с использованим спектрального признака устойчивости, установлены условия устойчивости для расчёта течений со специальными параметрами и для данного типа геометрии трубы. Критерии устойчивости представлены с учётом параметров модели.

Ключевые слова: система Навье – Стокса, граничные условия, сужение сечения трубы, перетекание жидкости, численный метод, спектральный признак устойчивости.

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Introduction

In the study of actual piping systems, which can be considered as the element of unit transport system, one of the most important issues of control of their technical condition is to detect small leaks of products transported, as they cause substantial economic losses and threats to the environment. The investigaton of flow in the pipeline with narrowing section is very important too/ This problems are solved by experimental methods [1-7]; however, a significant interest lies in the question of the use of mathematical methods for modeling of these processes. These methods make it possible to explore a wide class of phenomena and processes, which are accompanied by small leakage products. In some cases [8, 9], it is possible to obtain analytical solutions to the problem of fluid flow in the regions with different geometric configurations, but in most practical problems, it is necessary to use numerical methods for solving the issues of such kind. This is because the real technical systems have a complex spatial configuration, and often correct formulation of boundary conditions is either impossible or can be made only in approximate form and size of the area in which the researches require the construction of difference schemes with fixed parameters of the significant volume calculations nets. For a long time, this problem could not be resolved due to the limited capabilities of the computer systems. Modern computing capabilities allow the implementation of sophisticated mathematical models that use a system of Navier-Stokes equations to solve practical problems for engineering systems [10, 11]. The actual problem of estimating parameters of the flow, the universality of mathematical models, and models of viscous fluid flow in particular allows to apply them not only to study of the type of trunk pipeline systems, which is the main part of the proposed article, but also for diagnosing the origins of technological pipelines for different purposes used in cars, reactors, utilities, etc. In this paper, the model for flow in a pipeline with the presence of product leaks is constructed, which includes a system of equations, boundary conditions and the special conditions for the variation of pressure, and difference scheme for this system is implemented, the computational algorithm is build that examines the stability of difference schemes for a given performance liquid that is transported. It is possible to connect the industrial objects technical diagnostics problem with the problem of the corresponding mathematical models stability – as usual, diagnostics task can be formulated as mathematical model (equation, the system of equation etc.) with initial and boundary conditions, which are small disturbed - the great disturbances can lead to the loss of the model's adequacy [13]

Simulation of the Viscous Fluid Flow in the Pipe under the Steady Pressure Drop

The system of Navier-Stokes equations in primitive variables. When modeling the flow of a viscous fluid through pipes in the general formulation, the system of Navier-Stokes equations in a cylindrical coordinate system is used [8]:

$$\begin{cases} \dfrac{\partial v_r}{\partial t} + \vec{v}\nabla v_r - \dfrac{v_\varphi}{r} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \nu\left(\Delta v_r - \dfrac{v_r}{r^2} - \dfrac{2}{r^2}\dfrac{\partial v_\varphi}{\partial \varphi}\right); \\ \dfrac{\partial v_\varphi}{\partial t} + \vec{v}\nabla v_\varphi + \dfrac{v_r v_\varphi}{r} = -\dfrac{1}{\rho r}\dfrac{\partial p}{\partial \varphi} + \nu\left(\Delta v_\varphi - \dfrac{v_\varphi}{r^2} + \dfrac{2}{r^2}\dfrac{\partial v_r}{\partial \varphi}\right); \\ \dfrac{\partial v_z}{\partial t} + \vec{v}\nabla v_z = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \nu\Delta v; \\ \dfrac{1}{\rho}\dfrac{\partial r v_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial v_\varphi}{\partial \varphi} + \dfrac{\partial v_z}{\partial z} = 0, \end{cases} \quad (1)$$

where $\nabla f = \vec{i}_r \dfrac{\partial f}{\partial r} + \vec{i}_\varphi \dfrac{1}{r}\dfrac{\partial f}{\partial \varphi} + \vec{i}_z \dfrac{\partial f}{\partial z}$,

$\Delta f = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 r}{\partial \varphi^2} + \dfrac{\partial^2 f}{\partial z^2}$ – accordingly is the operator gradient and operator Laplacian in the cylindrical coordinate system. The system (1) can be written in a simpler form by the following assumptions:

– from a three-dimensional cylindrical coordinate system (1) a transition to a two-dimensional Cartesian system - one of the coordinates is chosen according to the pipe length, and the other – according to its cut;

– the behavior of the flow in a rectangular region in the presence of defects is studied; they are modeled as zones of leakage through the surface of the pipe;

– a three-dimensional effects are not taken into the account – similar to Poiseuille flow, and the transient nature of the process is not taken into the account as well.

In this case, the system of Navier-Stokes equations is written in the following form:

$$\begin{cases} u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\cdot\dfrac{\partial p}{\partial x} + \nu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right); \\ u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = -\dfrac{1}{\rho}\cdot\dfrac{\partial p}{\partial y} + \nu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right); \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0. \end{cases} \quad (2)$$

It is believed that the pressure along the length of the pipe varies linearly:

$$p = p_0 - kx, \quad (3)$$

where k – coefficient of pressure drop. The method of solving the Navier-Stokes equations, which is used in the implementation of the components of the velocity vector u and v, is called the method of solution using primitive variables [10]. The assumption (3) corresponds to the real cause of liquid transportation in industrial pipelines.

Boundary conditions. It is believed that the current area – rectangle, generally speaking, is of infinite length – thus, the cylindrical pipe is modeled; x – longitudinal coordinate, y – transverse, in fact, along the pipe diameter.

$$\begin{cases} u\big|_{x=0} = -\dfrac{ky^2}{4\mu} + \dfrac{kRy}{2\mu}; \\ u\big|_{y=0} = u\big|_{y=2R} = 0; \\ v\big|_{x=0} = v\big|_{y=0} = 0; \\ v\big|_{y=2R} = \begin{cases} 0,\ x \le x_1, x \ge x_2; \\ V_{leak}, x \in [x_1, x_2], \end{cases} \end{cases} \qquad (4)$$

where $[x_1, x_2]$ – piece of leakage, arbitrary configuration of leaks zones in length and intensity sources is possible; μ – dynamic viscosity of the product transported; R – pipe radius; ρ – product density. It is believed that when $x = 0$, the flow of fluid through the pipe is accurately described by the Poiseuille flow model [8]; $\mu/\rho = \nu$ – kinematic viscosity; V_{leak} – rate of fluid leakage through the pipe surface, which can change the sign depending on the type overflow.

Difference schemes for the Navier-Stokes system

The solution of (2) with boundary conditions (3) is performed by the method of finite differences, where variables are selected: Δx – step in longitudinal coordinate, Δy – step in transverse coordinate. Δx and Δy can be changed in time or in space/ To replace their original counterparts, the following difference-based dependencies are used [12]:

$$\begin{aligned} &\frac{\partial u}{\partial x} = \frac{u_m^{n+1} - u_m^n}{\Delta x}, \frac{\partial v}{\partial x} = \frac{v_m^{n+1} - v_m^n}{\Delta x}; \\ &\frac{\partial u}{\partial y} = \frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y}, \frac{\partial v}{\partial y} = \frac{v_{m+1}^{n+1} - v_{m-1}^{n+1}}{2\Delta y}; \\ &\frac{\partial^2 u}{\partial x^2} = \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}, \frac{\partial^2 v}{\partial x^2} = \frac{v_m^{n+1} - 2v_m^n + v_m^{n-1}}{\Delta x^2}; \quad (5) \\ &\frac{\partial^2 u}{\partial y^2} = \frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta y^2}, \frac{\partial^2 v}{\partial y^2} = \frac{v_m^{n+1} - 2v_m^n + v_m^{n-1}}{\Delta y^2}, \end{aligned}$$

difference analogues (4) are used to the system (2), the following difference equation analogues are received:

$$\begin{aligned} &u_m^n \frac{u_m^{n+1} - u_m^n}{\Delta x} + v_m^n \frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y} = -\frac{1}{\rho}k + \\ &+\nu\left(\frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{\Delta y^2}\right); \\ &u_m^n \frac{v_m^{n+1} - v_m^n}{\Delta x} + v_m^n \frac{v_{m+1}^{n+1} - v_{m-1}^{n+1}}{2\Delta y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \qquad (6) \\ &+\nu\left(\frac{v_m^{n+1} - 2v_m^n + v_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{v_{m+1}^{n+1} - 2v_m^{n+1} + v_{m-1}^{n+1}}{\Delta y^2}\right); \\ &\frac{u_m^{n+1} - u_m^n}{\Delta x} = -\frac{v_{m+1}^{n+1} - v_{m-1}^{n+1}}{2\Delta y}. \end{aligned}$$

With the aim of taking into the account all of the system equations after the pressure value is given by (3) and is a known as a function of the longitudinal coordinate, the third equation of system (6) is substituted into the second one, and then it acquires the following form:

$$\begin{aligned} &u_m^n \frac{v_m^{n+1} - v_m^n}{\Delta x} - v_m^n \frac{u_m^{n+1} - u_m^n}{2\Delta x} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \\ &+\nu\left(\frac{v_m^{n+1} - 2v_m^n + v_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{v_{m+1}^{n+1} - 2v_m^{n+1} + v_{m-1}^{n+1}}{\Delta y^2}\right). \end{aligned}$$

Realization of computational algorithm

Equations of the system (6) are written as a system of equations with a three diagonal matrix, that allows the application of the sweep method for its decision [12]:

$$\begin{aligned} &u_{m-1}^{n+1}\left(-\frac{v_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}\right) + u_m^{n+1}\left(\frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}\right) + \\ &+u_{m+1}^{n+1}\left(\frac{v_m^{n+1}}{2\Delta y} - \frac{\nu}{\Delta y^2}\right) = -\frac{1}{\rho}k + \frac{\left(u_m^n\right)^2}{\Delta x} + \nu\left(\frac{u_m^{n-1} - 2u_m^n}{\Delta x^2}\right); \\ &v_{m-1}^{n+1}\left(-\frac{\nu}{\Delta y^2}\right) + v_m^{n+1}\left(\frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}\right) + v_{m+1}^{n+1}\left(-\frac{\nu}{\Delta y^2}\right) = \qquad (7) \\ &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\left(u_m^n\right)\left(v_m^n\right)}{\Delta x} + \nu\left(\frac{v_m^{n-1} - 2v_m^n}{\Delta x^2}\right) + v_m^n\left(\frac{u_m^{n+1} - u_m^n}{\Delta x}\right). \end{aligned}$$

The coefficients of the system of equations with a three diagonal matrix are written in the following way:

$$d_m^1 = s - \frac{v_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}; \quad e_m^1 = \frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2};$$

$$f_m^1 = \frac{v_m^n}{2\Delta y} - \frac{\nu}{\Delta y^2}; \qquad (8)$$

$$b_m^1 = -\frac{1}{\rho}k + \frac{(u_m^n)^2}{\Delta x} + \nu\left(\frac{u_m^{n-1} - 2u_m^n}{\Delta x^2}\right)$$

– for component u,

$$d_m^2 = -\frac{\nu}{\Delta y^2}; \quad e_m^2 = \frac{u_m^n}{\Delta x} - \frac{\nu}{\Delta x^2} + \frac{2\nu}{\Delta y^2}; \quad f_m^2 = -\frac{\nu}{\Delta y^2};$$

$$b_m^2 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\left(u_m^n\right)\left(v_m^n\right)}{\Delta x} + \nu\left(\frac{v_m^{n-1} - 2v_m^n}{\Delta x^2}\right) + \qquad (9)$$

$$+ v_m^n\left(\frac{u_m^{n+1} - u_m^n}{\Delta x}\right)$$

– for component v.

It is necessary to keep the information about u and v values on two layers on a coordinate of x: $n-1$ and n; values of speed on a layer $n+1$ are determined in the process of solving the problem. For formulate the problem in more compact form an overhead index is cast aside, whereupon we receive:

$$d_m u_{m-1} + e_m u_m + f_m u_{m+1} = b_m.$$

Taking into account the standard assumption about linearity of dependence between the components u_{m-1} and u_m we receive:

$$u_{m-1} = k_m u_m + L_m;$$
$$d_m k_m u_m + d_m L_m + e_m u_m + f_m u_{m+1} = b_m;$$
$$u_m = -\frac{f_m}{d_m k_m + e_m}u_{m+1} + \frac{b_m - d_m L_m}{d_m k_m + e_m}; \qquad (10)$$
$$k_{m+1} = -\frac{f_m}{d_m k_m + e_m}; \quad L_{m+1} = \frac{b_m - d_m L_m}{d_m k_m + e_m}.$$

For solving the system (6) with boundary conditions (3) the sweep method was twice realized, thus, the form of record of boundary conditions allows to take into account the localization of places of liquid source. Direct and reverse sweep motion allowed to define *u* and *v* on the layer of *n+1*, and after the proper reassignment of values on layers an algorithm repeats oneself. The amount of steps of iterative process is determined by the stability of iterative procedure and necessity of pipeline control of specified length.

Stability Investigation of the Difference Scheme

Stability investigation of difference schemes is carried out by the spectral feature of stability [12]. Since the first two equations (6) are almost identical, the stability investigation is carried out only for the first equation. The peculiarity of the spectral features is that the Navier-Stokes system is non-linear, so it is necessary to use the principle of frozen coefficients, whereby the coefficients of the system (6) must be constant. With the implementation of frozen coefficients method, the component u_m^n is written as:

$$u_m^n = \lambda^n e^{i\varphi}. \qquad (11)$$

The stability condition of difference scheme can be written as:

$$|\lambda| \le 1.$$

For the first equation of (5):

$$u_m^n \frac{u_m^{n+1} - u_m^n}{\Delta x} + v_m^n \frac{u_{m+1}^{n+1} - u_{m-1}^{n+1}}{2\Delta y} =$$
$$= \nu\left(\frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta x^2}\right) + \nu\left(\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{\Delta y^2}\right),$$

using the notation:

$$\frac{\Delta x}{\Delta y} = p; \; u_m^n \approx U = A; \; \frac{v_m^n}{2} \approx V = B; \; \frac{\nu}{\Delta x} = C, \quad (12)$$

the studied equation takes the form:

$$A\left(u_m^{n+1} - u_m^n\right) + Bp\left(u_{m+1}^{n+1} - u_{m-1}^{n+1}\right) =$$
$$= C\left(u_m^{n+1} - 2u_m^n + u_m^{n-1}\right) + p^2\left(u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}\right).$$

The use of characteristic values (12) allows the selection of those parameters of computational grid, which provide the stability of computational scheme. In the view of (11) and Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

derived by:

$$A\left(\lambda^2 - \lambda\right) - 2Bp\lambda^2 i\sin\varphi =$$
$$= C\left(\lambda^2 - 2\lambda + 1\right) - 4p^2\lambda^2\sin^2\frac{\varphi}{2}. \qquad (13)$$

The equation (13) is a quadratic one with the complex coefficients. Its roots are determined by the relations:

$$\lambda_1 = \frac{A - 2C + \sqrt{A^2 - 8BCpi\sin\varphi + 4Cp^2\sin^2\frac{\varphi}{2}}}{2(A - 2Bpi\sin\varphi - C + 4p^2\sin^2\frac{\varphi}{2})}, (14)$$

$$\lambda_2 = \frac{A - 2C - \sqrt{A^2 - 8BCpi\sin\varphi + 4Cp^2\sin^2\frac{\varphi}{2}}}{2(A - 2Bpi\sin\varphi - C + 4p^2\sin^2\frac{\varphi}{2})}. \quad (15)$$

In general, the test conditions $|\lambda_1| \le 1$, $|\lambda_2| \le 1$ requires finding the values $Re\lambda_1$, $Re\lambda_2$, $Im\lambda_1$, $Im\lambda_2$ and testing conditions:

$$\sqrt{(Re\lambda_1)^2 + (Im\lambda_1)^2} \le 1; \quad \sqrt{(Re\lambda_2)^2 + (Im\lambda_2)^2} \le 1, \quad (16)$$

requiring complex mathematical calculations. However, in this case, the research of the conditions (16) is carried out taking into account the characteristic process variables, namely:

$$B \approx 0, \quad C \approx 10^{-5}, \quad A \approx 1. \quad (17)$$

From the equation (14) and inequality (16) in such assumptions the following inequality implies:

$$(16 - \varepsilon)p^2 + 64p^4 \ge 0, \quad (18)$$

and from (15), (16) we receive:

$$-\sqrt{1 + \varepsilon p^2} \le 1 + 8p^2, \quad (19)$$

whereas $\varepsilon \approx 10^{-5}$. It is obvious that these inequalities are performed for all values p. For large values of leakage rate the difference scheme requires special research, not only in terms of the stability calculation; it is necessary to check the adequacy of the model (2) – (3) and its relevance to the physical picture of the process. While solving the current problems of the research for which the following conditions (17), (18) and (19) take place, the estimated grid parameters are chosen with the requirements of the accuracy of calculations, because the numerical scheme is absolutely stable. The calculations for the flows with different kinds of overflow and for the different level of sections narrowing have been made and gave the good agreement with the correspondent experimental data. The presented results was applied for the next problems solving:

- The flow with the low leakage in pipeline in the process of the oil transportation;
- The complex problem of the harmful substance distribution estimation in the zone of pipeline
- The flow in the drill string with the fluid injection and variable properties of liquid along the length of the tube;
- The flow of the viscous liquid in the pipeline with the substance deposits on the pipeline wall;
- The flow in the pipeline with the forced narrowing of the section

Simulation of the flow in the pipeline with defects through which the outflow of fluid is conducted for the following flow parameters, the pipe geometry, properties of liquids and gases, the linear pressure drop along the length of pipe:

− the average fluid velocity in the pipeline – 2-8 m/s, the average fluid velocity in the wells for oil extraction – 2-5 m/s;

− typical rate of low leakage and the overflow trough the surface – up to 50 cm/s;

− dynamic viscosity of the fluid (water) and transported technological liquids – 0,001 – 0,01 kg/cm;

− kinematic viscosity – 0,000001 – 0,00001 m^2/s;

− characteristics of pressure drop – $K = 0{,}064 - 0{,}096$;

− step on the longitudinal coordinate – 0,08 m;

– step on the transverse coordinate – 0,025 m, which corresponds to 1,25 m diameter pipe at 50 control points along the transverse coordinate, this step can be changed in the process of solution.

Conclusions

According to the results of the conductive simulation, the following conclusions can be drawn:

− the use of Navier-Stokes equations in the form of (2) with boundary conditions (4) allows to obtain an adequate physical model that can serve as a basis for the diagnostic algorithms construction of detecting the localization of small leaks in the pipeline. The condition (3) allows, on the one hand, to simplify the calculating algorithm; otherwise, the problem is reduced to the Poiseuille problem [8], it is necessary to set an initial approximation of the velocity field and the initial approximation of the pressure field, whereas, in this case, it is necessary to specify only conditions(4);

− the used difference schemes tested for stability: the case of flow parameters, which satisfy the condition (17), set the absolute stability of numerical schemes, however, require further research in the issues of sustainability schemes for which the condition (17) is not satisfied;

− the future investigation can be devoted to the studying of the correspondence between the stability of the difference scheme and the stability of the modelling moving of the real liquid, to the receiving the answer on the question – can we sort the numerical scheme instability and the instability of the modelling hydrodynamics process.

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