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APPLICATION OF DIFFERENTIAL EQUATION SYSTEMS TO SOCIO-ECOLOGICAL PROCESS RESEARCH

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Abstract. The article examines the application of mathematical modeling methods for analyzing socio-ecological systems based on a system of linear differential equations with constant coefficients. The proposed model describes the interaction among three key factors: regional population size, environmental pollution levels, and the state of the flora. The Lotka–Volterra approach is employed to construct the system, allowing for the investigation of solution stability and the system's behavior in response to initial conditions and parameters. The methodology involves determining the model coefficients through expert assessments combined with Kendall's concordance criterion, ensuring the

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reliability of the obtained results. Examples of using statistical data are provided, along with explanations of how modifications to initial conditions influence the stability of socio-ecological systems. Special attention is given to challenges arising from stiff systems, which are caused by significant differences in the eigenvalues of the coefficient matrix. To address this issue, a coefficient normalization approach is proposed. The results are of practical importance for predicting the dynamics of socio-ecological processes and supporting decision-making regarding sustainable natural resource management. The proposed model can be applied to evaluate ecological balance, develop pollution reduction strategies, and ensure the sustainable development of regional systems.

Keywords: socio-ecological systems, mathematical modeling, system of differential equations, Lotka–Volterra model, stability, expert assessment, Kendall criterion.

Introduction

In the simulation of the behavior of ecological and social systems of widespread use, methods of mathematical modeling and, in particular, methods of ordinary differential equations have been applied. The most historically high priority is the work of Lotka and Volterra [2,7], and also, differential models are given in [1, 3].

The main task is to construct the models which give the adequate possibility to describe the processes, one's quality characteristics. It is the first step of such processes investigations, future researching will be connected with the applying statistical data to receive the real characteristics of studying processes. So, such article has the model and theoretical character, which consider the theoretical models.

Another questions is to consider the possibility of applying the system of linear differential equations with the constant coefficients to design the practical processes. It is considered the main theoretical conceptions from the point of view of adoption to solve the practical problems.

The possibilities for describing real objects using systems of ordinary differential equations are considered, among be ass of linear systems with constant coefficients is allocated. This allows us to build relatively simple models that really reflect the picture of phenomena and systems being modeled.

Purpose of the Study

The purpose of this study is to develop a mathematical model of a socio-ecological system based on linear differential equations to analyze the interaction between key environmental factors and determine the conditions for the sustainable development of the region. The proposed model enables the assessment of ecological balance, prediction of dynamic changes in key indicators, and supports decision-making processes regarding the sustainable use of natural resources.

Analysis of Modern Foreign and Domestic Research and Publications

The historical background of modeling the interaction between population–environment–biota is closely related to the Lotka–Volterra approach and the subsequent development of differential models in applied studies. In the context of this work, systems of ordinary differential equations (ODEs) with constant coefficients are used as the basic tool for describing dynamics and analyzing the stability of socio-ecological systems. This is emphasized in the Introduction with references to the classical works [2] and methodological sources [1, 3], which form the methodological foundation of the modeling approach.

Among modern international approaches, several trends are particularly significant:

– Agent-based (individual-based) modeling to reproduce micro-level behavior and its macro-level consequences [5];

– Integrated frameworks for global sustainability that combine social and natural subsystems [6];

– Studies of critical transitions and tipping points in complex socio-ecological systems [7];

– The Ostrom framework for analyzing the sustainability of SES [8];

– Comparative reviews of modeling approaches for integrated environmental assessment (system dynamics, ABM, ODE-based models, etc.) [9];

– Theoretical foundations of adaptive capacity and resilience [10];

– The panarchy theory as a multiscale approach to ecosystem–society transformations [11];

– New challenges and opportunities for modeling coupled human–natural systems [12].

These studies define the modern context and justify the combination of linear ODE models with scenario-based or expert-driven parameter calibration.

Domestic contributions are represented mainly by methodological works in higher mathematics and experimental planning [1, 3], as well as general principles of scientific research [4], which provide the mathematical tools for model construction, identification, setting initial conditions, and validating data consistency. Combined with the expert-based approach, these works create a foundation for parameterizing socio-ecological models under conditions of limited observations.

A separate area involves methods for determining model coefficients based on expert assessments with consistency control using Kendall's concordance coefficient. The article provides the corresponding formula and practical insights regarding threshold values, which make it possible to validate the calculated weights and avoid issues related to stiff systems by scaling the parameters. This directly supports the approach proposed by the authors for model identification and stability analysis.

Thus, existing international frameworks (SES-framework, panarchy, tipping points, ABM) complement the chosen toolkit of linear ODEs and justify the use of expert-oriented parameter identification for practical regional applications, while domestic methodological sources provide the formal mathematical and experimental foundations required for effective modeling.

Methods

The system described by three differential equations with respect to functions is considered: x (t) - population in the region; y (t) - level of pollution and other harmful effects on the environment caused by population activity; z (t) - the level of the flora of the region (trees, forests, gardens, etc.). We write down a system of differential equations that describes the change in the time of the functions x (t), y (t), and z (t).

We use systems of linear differential equations with constant coefficients. Change in x (t) per unit of time is written as an equation.

$$\frac{dx}{dt} = Ax - By + Cz. \quad (1)$$

Equation (1) can be interpreted as follows: the population of the region increases (decreases) in proportion to x (t) ($A > 0$ increases, $A < 0$ decreases); decreases in proportion to the level of pollution and other harmful effects of human activity y (t) $B > 0$: increases proportionally to the magnitude of the character of the flora (green plantations, forests, orchards, cities of the region, etc.); $C > 0$. The coefficients A, B and C of the simplest model are considered constant, in more complex models they are considered functions of time, determined experimentally. The speed of the change y (t) can be described as follows (taking into account model [3]):

$$\frac{dy}{dt} = Dx - Ez, \quad (2)$$

which can be commented as follows: the speed of the level of harmful effects changing on the environment is proportional to the population in the region (coefficient D take into account both the vital activity of the population and the results of the functioning of industrial objects of the region), $D > 0$, and have a decreasing depends on flora condition region – as already mentioned (forests, gardens, garden cultures, etc.) $E > 0$.

The speed of change for z (t) can be described using the third differential equation

$$\frac{dz}{dt} = Hx - Gy + Fz, \quad (3)$$

that takes into account the following formulas: (the qualitative quantity of the characteristic of the flora condition $z(t)$ depends on the following factors: population size $x(t)$. In this case, when choosing the values and the magnitude of the coefficients H the level of environmental consciousness of the population should be taken into account, at a high level, when worrying about the state of the land plots by the population compensates for the regulation of its life and the functioning of industrial enterprises H > 0, at a low level (little attention is paid ignorance of ecology.) H < 0 for other factors on the change z (t) is the level of pollution y (t), obviously G > 0, as well as from the real state of the flora z (t), F > 0. Taking to account this third equation it is possible so the system can be written in the form :

$$\begin{cases} \frac{dx}{dt} = Ax - By + Cz \\ \frac{dy}{dt} = Dx - Ez \\ \frac{dz}{dt} = Hx - Gy + Fz \end{cases} \quad (4)$$

Such system can be solved using the analytical solution without the applying of numerical methods.

For the correct problem statement of modeling, it is necessary to set initial conditions

$$\begin{cases} x(0) = x_0 \\ y(0) = y_0 \,, \\ z(0) = z_0 \end{cases} \quad (5)$$

which can be determined from statistical data on the population size, level of pollution, and the state of the flora of the region. It should be noted that in the system (4), it is necessary to clearly define in which units the characteristics x (t), y (t), z (t) are defined to understand the coherence of the coefficients. One way of resolving this point is to choose monetary equivalents for x(t), y (t), and z (t) – for example, losses from environmental pollution by harmful products, and estimating the profit from the amount and condition of green areas in the region. The system (4) with initial conditions (5) can be analyzed from the point of view of the possible behavior of its solutions. In particular, the desired option is to output the value of x (t), y (t), and z (t) to a certain stable level – from a mathematical point of view, it is desirable to fulfill the condition:

$$\begin{cases} \lim\limits_{t\to\infty} x(\mathrm{t}) = x_\infty; \\ \lim\limits_{t\to\infty} y(\mathrm{t}) = y_\infty; \\ \lim\limits_{t\to\infty} z(\mathrm{t}) = z_\infty. \end{cases} \quad (6)$$

The right-hand sides of (5) and (6) are different – the state of x (t), y (t), z (t), is determined by the initial conditions, but does not equal this initial value. The analysis of the system (4) with the conditions (5) allows us to establish the mathematical relation on the matrix coefficient of the equations system (4), for which the given solutions will be as shown in Fig.1 To this end, we write the characteristic equation of the system (4) [3]:

$$\det \begin{vmatrix} A-\lambda & -B & C \\ D & -\lambda & -E \\ H & -G & F-\lambda \end{vmatrix} = 0 \quad (7)$$

which is equivalent

$$\lambda(\lambda - F)(A-\lambda) - CDG + BEH + \\ + CH\lambda - EG(A-\lambda) + DB(F-\lambda) = 0\,. \quad (7')$$

It is necessary to consider all possible variant of solution (7') – three real roots or one real and two complex. The case shown in Fig. 1 corresponds, for example, to the following condition: one of the three roots of the equation (7) is equal to zero, and the other two - either valid negative or integral with a negative valid part:

$$\begin{cases} \lambda_1 = 0 \\ \lambda_2 < 0\,; \\ \lambda_3 > 0 \end{cases} \quad \begin{cases} \lambda_1 = 0 \\ \mathrm{Re}\,\lambda_2 < 0\,. \\ \mathrm{Re}\,\lambda_3 > 0 \end{cases} \quad (8)$$

in the case of (4) x (t), y (t), z (t) at a rate satisfying condition (5), monotonically increasing or decreasing.

$$\begin{cases} \lambda_1 = \lambda_2 = 0 \\ \lambda_3 < 0 \end{cases}. \quad (9)$$

The values in (24), (25) are also functions of the initial conditions (5) and the coefficients of the system (4).

However, case (9) is not guaranteed to cause the condition of stabilization (6), therefore the main option guaranteeing fulfillment (6) is a set of conditions (8).

$$\lambda^3 + \lambda^2(A+F) - \lambda(FA - CH - GE + DB) + \\ + (BDF - EGA + BEH - CDG) = 0\,; \quad (10)$$

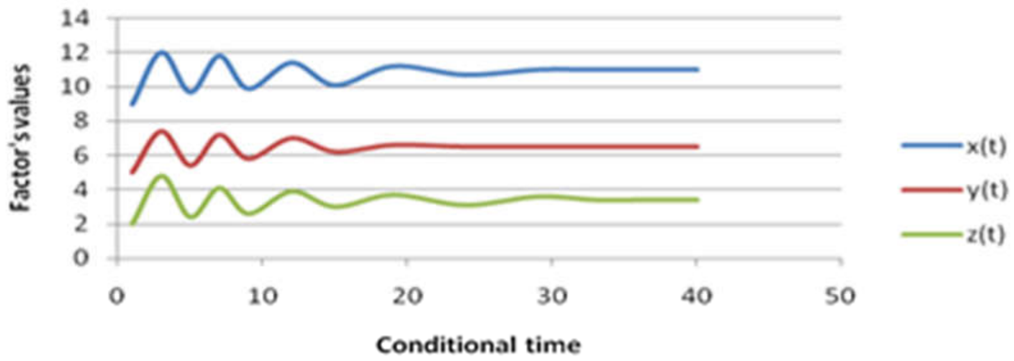

Figure 1 – Distributions of the factor conditional values

$$BDF\text{-}EGA+BEH\text{-}CDE=0\,. \qquad (11)$$

Then it can be look like:

$$\lambda^3+\lambda^2(A+F)- \\ -\lambda(FA-CH-GE+DB)=0. \qquad (12)$$

$\lambda=0$, a to find the other two roots we have an equation:

$$-x^2+(A+F)x-(FA-CH-GE+DB)=0. \qquad (13)$$

Then

$$\lambda_{2,3}=\frac{-(A+F)\pm\sqrt{(A+F)^2-4(FA-CH-GE+DB)}}{-2}= \\ =\frac{(A+F)\mp\sqrt{(A+F)^2-4(FA-CH-GE+DB)}}{2}. \qquad (14)$$

If

$$D=(A+F)^2-4(FA-CH-GE+DB)<0, \qquad (15)$$

to $\lambda_{2,3}$ – complex roots and for use of the second of the conditions (8) it is necessary to:

$$\operatorname{Re}\lambda_2=\operatorname{Re}x_3=\frac{A+F}{2}<0. \qquad (16)$$

Thus, for case (15) for stabilization *x(t), y(t), z(t)* (fig.1) it is necessary to fulfill the conditions (11) and (15), (16). For the case:

$$D=(A+F)^2-4(FA-CH-GE+DB)>0. \qquad (17)$$

To fulfill the first conditions (8) it is necessary to:

$$\begin{cases} A+F+\sqrt{D}<0 \\ A+F-\sqrt{D}<0 \end{cases} \qquad (18)$$

Conditions (18) give the demanding of receiving the solutions with the properties of stability, it is very important for finding the limited solutions.

Therefore, in this case, to fulfill the condition of stabilization x(t), y(t), z(t) the conditions are necessary.

For case (9) it is necessary that in the equation (10) the free term be equal to zero (condition (11)), and also the coefficient at X:

$$FA-CH-GE+DB=0\,. \qquad (19)$$

Then equation (10) takes the form of:

$$-\lambda^3+\lambda^2(A+F)=0,$$

So:

$$-\lambda+F+A=0. \qquad (20)$$

So, to fulfill condition (9) it is necessary that:

$$F+A<0. \qquad (21)$$

However, case (9) provides for the stimulation of functions

$$x(t), y(t), z(t) \qquad (22)$$

look like:

so:

$$\begin{cases} x(t)=a_1+b_1t+c_1e^{\lambda t} \\ y(t)=a_2+b_2t+c_2e^{\lambda t} \\ z(t)=a_3+b_3t+c_3e^{\lambda t} \end{cases} \qquad (23)$$

and there is an additional condition either:

$$b_1=b_2=b_3=0 \qquad (24)$$

or

$$b_1<<1; b_2<<1; b_3<<1\,. \qquad (25)$$

Results and models accuracy theoretical research

The practical implementation of the published models requires data on the values of the coefficients of the matrix of systems (4). To solve this problem, either static data on the relationship between the corresponding values or the method of expert assessments, which in this case may be the next among a wide range of experts, is used.

The schedule can look like in the figure 1.

The following poll is conducted: they are asked to rank numbers in order of importance for the coefficients, as shown in Table 1:

Table 1 – Importance of mutual influence, between values x(t), y(t), z(t)

	E_1	E_2	E_3	…	E_{n-1}	E_n	$\sum$
A	2	1	2		8	3	X_1
B	5	3	5		7	1	X_2
C	3	8	4		5	4	X_3
D	6	5	7		4	2	X_4
E	1	2	8		1	6	X_5
F	7	7	1		3	5	X_6
G	8	6	3		2	7	X_7
H	4	4	6		6	8	X_8

After conducting this survey for each of the coefficients A, B, C, D, E, F, G, H, is its total score Xi; and i = 1,…8, and the value of the corresponding coefficient, for example, of the coefficient D, is obtained by the formula:

$$D = \frac{x_4}{\sum_{i=1}^{8} x_i} \quad . \tag{26}$$

This allows us to obtain the values of the coefficients of the matrix of system (4) in the form of numbers distributed on the segment [0; 1]:

$$A = \frac{x_1}{c}; B = \frac{x_2}{c}; C = \frac{x_3}{c}; D = \frac{x_4}{c};$$
$$E = \frac{x_5}{c}; F = \frac{x_6}{c}; G = \frac{x_7}{c}; H = \frac{x_8}{c}, \tag{27}$$

where c=$\sum_{i=1}^{8} x_i$.

After determining the coefficients (27), the implementation of the stabilization conditions is checked. If the above conditions are not met, then changes in the coefficients are made, or conducting surveys for a wider range of experts. In the analysis of the results obtained by the given values of the elements of the matrix of system (4), if necessary, the initial conditions (5) are mixed to obtain the desired values (6).

In fact, this means that the necessary values of the initial conditions that guarantee the stable development of the socio-economic system are established.

The Kendall criterion is used to control the consensus of experts: the coefficient of concordance is calculated by such formula:

$$K = \frac{12S}{m^2\left(n^3 - n\right)}, \tag{28}$$

where

$$S = \sum_{i=1}^{n}\left(d_i - \overline{d}\right)^2, \tag{29}$$

$$d_i = \sum_{j=1}^{m} v_{ij},$$

v_{ij} – range of the *i*-th factor in the j-th expert, d_i – the sum of ranks *i*-th factor in all *m* experts;

n – number of indicators; *m* is the number of experts.

$\overline{d}$ – the average level of all criterion.

If K is greater than 0.75, then the consistency in the indicators of experts is considered satisfactory [5]. Such a method is called the expert assessment method

It should be noted that there is a case in which the elements of the matrix of system (4) are unquestionable in relation to their values for example, the value of the coefficient D can be deduced from Table 1, since it can be directly determined from statistical data. In this case, the following method is proposed to establish the real values of the coefficients: the coefficients that can be determined by the methods are deduced from the table. In this case, in Table 1, the number of rows is reduced, and then the implementation of the methodology for evaluating the importance is implemented. In order to ensure the adequacy of the model and the same order of values of the coefficients, all other coefficients defined on the interval [0; 1] are scaled by the magnitude of the coefficient, which is removed from the table by the formula:

$$K = \tilde{A} * K_i, \tag{30}$$

where K_i – the value of the coefficient on the segment [0; 1];

A – the value of the coefficient, which is determined without the application of the expert estimation method. This avoids the problems associated with the peculiarities of rigid systems of differential equations, which are characterized by a significant spread between the values of the eigenvalues of the matrix of the system, and, as a result, a significant spread in the rate of change of functions, which are solutions of the system [4]. For example, considering the simplest simulated system:

$$\begin{cases} \frac{dx}{dt} = 100x \\ \frac{dy}{dt} = 0,001y \end{cases} \quad (31)$$

the solving of which is:

$$\begin{cases} x = x_0 e^{100t} \\ y = y_0 e^{0,01t} \end{cases} \quad (32)$$

Obviously, these functions have different growth rates - for the same controlled time of time [0; 5], the function y (t) does not change at all while x (t) exponentially grows (like e^{100t}). The approach described in the Table 1, with the use of (26) and (27), allows avoiding the problems associated with the rigidity of systems. If some elements of the matrix of the system (4) are determined experimentally, and at the same time they have a significant spread, then to obtain an objective model, it is necessary to use such methods and methods of processing experimental data as the method of reference to the experiment and the method of Himmelblau [6], discarding abnormal values. However, this situation is not specific to real socio-ecological systems. Typically, in such cases, we are aware of several characteristics, and a scaling coefficient system (4) should be used to maximize the ratio.

Conclusions

It was shown that the method of systems linear differential equations with the constant coefficients applying allows to construct the models of real socio-ecological system. It was shown that the applying of the methods, which based on the systems of ordinary differential equation allows to receive the solutions, which satisfied the condition of stability – very important moment to real processes in socio- economical systems and by the using the presented method of model coefficient definition to receive the results which demonstrate the good agreement in describing the behavior of real processes.

To check the model's adequacy the method of expert marks is used; it allows to defined the model's coefficients value which correspond to real process and systems characteristics;

The Kendall criterion allows to show the good agreement of expert's point of view on the tested processes.

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Conflict of interest

None.

References

1. Pak V. V., Nosenko Yu. L. Higher Mathematics. D. Stekler, 1997. 560 p. [in Ukrainian]
2. Hairer E., Norsett S., Wanner G. The Solution of Ordinary Differential Equations. Springer Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo. 1997. 572 p.
3. Anistratenko V. O., Fedorov V. G. Mathematical Planning of Experiments in APC. Kyiv: Higher School, 1993. 374 p. [in Ukrainian]
4. Doroshenko V. M. Fundamentals of Scientific Research. Kyiv: ISDO, 1993. 128 p. [in Ukrainian]
5. Grimm V., Railsback S. F. Individual-Based Modeling and Ecology. Princeton University Press, 2013.
6. Liu J., Dietz T., Carpenter S. R., et al. Systems integration for global sustainability. *Science*. 2015. Vol. 347(6225). 1258832. doi: 10.1126/science.1258832
7. Dakos V., et al. Ecosystem tipping points in complex socio-ecological systems. *Nature Ecology & Evolution*. 2019. Vol. 3(3). P. 355–362. doi: 10.1038/s41559-019-0793-7
8. Ostrom E. A general framework for analyzing sustainability of socio-ecological systems. *Science*. 2009. Vol. 325(5939). P. 419–422. doi: 10.1126/science.1172133

9. Schlüter M., et al. Challenges and opportunities for modeling human-natural systems. *Ecological Modelling*. 2020. Vol. 43. 109262. doi: 10.1016/j.ecolmodel.2020.109262

10. McGinnis M. D., Ostrom E. Social-Ecological Systems Framework: A multi-step methodological guide. *Ecology & Society*. 2023. Vol. 27(4). URL: https://ecologyandsociety.org/vol27/iss4/art39

11. Froese R., Andrino A., et al. Describing complex interactions of social-ecological systems for tipping point assessments: An analytical framework. *Frontiers in Climate*. 2023. Vol. 5. doi: 10.3389/fclim.2023.1145942

12. Milkoreit M., Hodbod J., Baggio J. A., et al. Defining tipping points for social-ecological systems scholarship – an interdisciplinary literature review. *Environmental Research Letters.* 2022. Vol. 17(3). doi: 10.1088/1748-9326/aaaa75

13. Rounsevell M. D. A., Robinson D. T., Murray-Rust D. From actors to agents in socio-ecological systems models. *Philosophical Transactions of the Royal Society B: Biological Sciences*. 2012. Vol. 367(1586). P. 259–269. doi: 10.1098/rstb.2011.0187

14. Schulze J., Müller B., et al. Agent decision-making: The elephant in the room — enabling the justification of decision model fit in social-ecological models. *Environmental Modelling & Software*. 2023. Vol. 165. 105759. doi: 10.1016/j.envsoft.2023.105759

ЗАСТОСУВАННЯ СИСТЕМ ДИФЕРЕНЦІАЛЬНИХ РІВНЯНЬ У ДОСЛІДЖЕННІ СОЦІО-ЕКОЛОГІЧНИХ ПРОЦЕСІВ

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Анотація. У статті розглянуто застосування методів математичного моделювання для аналізу соціо-екологічних систем на основі системи лінійних диференціальних рівнянь зі сталими коефіцієнтами. Запропонована модель дозволяє описати взаємодію між трьома основними факторами: чисельністю населення регіону, рівнем забруднення довкілля та станом флори. Для побудови системи застосовано підхід Лотки-Вольтерри, що забезпечує можливість дослідження стійкості рішень та поведінки системи залежно від початкових умов і параметрів.

Методика передбачає визначення коефіцієнтів моделі за допомогою експертних оцінок із застосуванням критерію узгодженості Кендалла, що гарантує достовірність отриманих результатів. Наведено приклади використання статистичних даних та пояснено, як модифікація початкових умов впливає на стабільність соціо-екологічних систем. Окрему увагу приділено проблемам жорстких систем, що виникають при значному розходженні власних значень матриці коефіцієнтів, та запропоновано способи їх уникнення за допомогою нормалізації коефіцієнтів. Отримані результати мають важливе практичне значення для прогнозування динаміки соціо-екологічних процесів та прийняття управлінських рішень щодо оптимізації використання природних ресурсів. Запропонована модель може застосовуватися для оцінювання екологічної рівноваги, планування заходів зі зменшення рівня забруднення та підтримки стійкого розвитку регіону.

Ключові слова: соціо-екологічні системи, математичне моделювання, система диференціальних рівнянь, модель Лотки-Вольтерри, стабільність, експертні оцінки, критерій Кендалла.